THE

"SCHWEDLER" BRIDGE.

A COMPARISON OF

THE VARIOUS FORMS OF GIRDER BRIDGES,

SHOWING THE

ADVANTAGES OF THE "SCHWEDLER" BRIDGE;

TOGETHER WITH

AN ELUCIDATION OF THE THEORETICAL PRINCIPLES OF THE SAME.

BEING A PAPER READ AT THE INSTITUTION OF CIVIL ENGINEERS,
AND TO WHICH THE COUNCIL AWARDED A MILLER PRIZE.

BY

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THE "SCHWEDLER" BRIDGE.

THE subject of the present paper being a comparison of the various systems of iron girder bridges, with particular reference to those constructed on the principle suggested by Herr Baurath Schwedler, it will be necessary to mention what other systems have been carried out or suggested. This will, perhaps, be most effectually done in an historical form, which will best illustrate the development of the theory of their construction.

Between 1830 and 1840 iron was first used in the form of beams, mostly for smaller bridges. This was found to be the shortest and cheapest way of putting up such bridges, a result very important in consequence of the immense development of railways at this time. Many girders of this sort were cast for the York, Midland Counties, Northern, and Eastern Railway Companies. They were composed of long castings held together by wrought-iron bands. Common sections are shown in Figs. 1 and 2, Plate 1. The second form was used where the admissible height of girder was very small. These bridges were copied both in Germany and France for the railways which began to spring up in those countries. Among the most important and largest of these was the cast-iron girder bridge over the Schelde at Ghent (1). It was erected by the engineers Marcellis and Duval, and consisted of two girders 60 feet long, which supported the road and at the same time served as parapet. Each girder consisted of two pieces, bolted together in the middle.

At about the same time appeared a kind of bridge on the Continent, the shape of which is exactly the opposite of that which theory would indicate as the proper form. The top flange is horizontal, the under one in the form of an arch, as shown in Fig. 3. These were constructed without any abutments capable of resisting horizontal thrust, and cannot be considered as arches,

but must be regarded as girders. The strains are therefore the greatest in the middle, where the moment of resistance is the smallest. These were soon found to be unserviceable and expensive, and were followed by a girder of the form of Fig. 4, in which the moment of strain is opposed approximately at every point of the girder by a moment of resistance equal to it; in other words, the beam is of equal strength throughout its length. Although these beams fulfil theoretical requirements, they were found, when employed for larger spans, not to last long, in consequence of the small resistance of cast iron to tension, especially when accompanied by continually repeated shocks, such as those caused by the frequent passage of railway trains. This gave rise to the idea of employing both cast and wrought iron in the construction of bridges, in such a manner that those parts exposed to tension should be made of wrought iron, and those exposed to pressure, of cast iron. A form of small span bridge is given in Fig. 5. One of the oldest of the larger bridges on this system is that designed by Stephenson in the year 1846. It is a road bridge over the North-Western Railway at Camden (2), as sketched in Fig. 6.

After the year 1846, the cast-iron parts in these and similar bridges were almost entirely replaced in the newer constructions by wrought iron, owing to the greater capability of resisting tension, and the much longer endurance of the latter under concussion, as shown by experience, and by the experiments made by Stephenson on the erection of the Britannia bridge. Notwithstanding this, in the bridges built on the principle suggested by Neville, and afterwards improved by Captain Warren, cast iron was employed for some time longer. A detailed description of these bridges will be unnecessary, as they are well known in England. It need only be remarked that, although the form of the Neville girder may be deduced from correct theoretical principles, the faults in its construction were numerous. Warren corrected many of these, and formed the upper member of his girder out of cast-iron tubes, the lower of wrought-iron bands. The struts between them were cast iron, the ties wrought iron. As an example may be cited the bridge over the Trent for the Great Northern Railway line (3).

In Germany, the system invented by Shifkorn followed the Neville system. His girder consisted of three long rails, one at the top and bottom, and one in the middle. These were bolted together at suitable distances, the four-cornered spaces between the bolts and rails being filled up with cross-formed iron castings. The latter were laid between the bands or rails, then the bolts tightened up till the whole became firm. These iron castings were all of one pattern, notwithstanding the various strains to which they are subject, and owing to the above-mentioned manner of bolting the parts together, an accurate calculation of the strains is impossible. This bridge was entirely condemned by the Austrian Society of Engineers.

Many bridges, composed partly of cast, partly of wrought iron, have been also constructed in America, most of the girder bridges being on the system of Long and Howe (which was first executed in wood). The names of Bollman, Whipple, and Fink ought also to be mentioned in connection with American bridges. They are, most of them, girder bridges, with upper and under horizontal flanges, some of them, however, being so trussed that the horizontal under boom may be left out. Those in which the upper member is parabolic, or of other similar form, have all double diagonals in each bay, whereas the Schwedler girder does not require this.

It would then appear that the combination of cast and wrought iron for bridges can be regarded as a period of transition in the European States, the girder bridges being now almost entirely constructed of wrought iron. In North America, on the contrary, such bridges are still very frequently built. The reason is probably not only that the North American cast iron is better able to resist tension, but also that the Americans make smaller requirements with regard to the duration of a bridge, than that it should be lightly, quickly, and cheaply put together.

In these few words on cast and cast and wrought-iron bridges, I have confined myself to girder bridges, that is to say, to such bridges which exercise no horizontal strain on the abutments, either tension or pressure. In the following notes on some of the systems for the construction of wrought-iron bridges which

have been carried out, I will also confine your attention to the same class of bridges, inasmuch as the "Schwedler" bridge is one of these.

In the year 1846, some of the foundry owners already began to assert that the upper boom of the girder, made of cast iron, could be much better, and just as cheaply, constructed of wrought iron, where the bridge had to carry heavy movable loads. In consequence of this view of the question, many bridges, more especially movable and drawbridges of all sorts, began to be built entirely of wrought iron. Fairbairn erected several bridges of this sort, also a large floating quay in Liverpool, the upper and under flange of the girder being in the box form. The experiments made on the construction of the Conway bridge, and of the railway bridge over the Menai Straits at Bangor (4), were of considerable importance for the construction of girders of this form, and for a further knowledge of the material used. Brunel suggested girders of a somewhat more convenient form, an example of which is the railway bridge over the Wye at Chepstow. The single vertical rib of Brunel's girder permitted a saving of material in comparison with the double walls of box-formed girder (see Fig. 7). The manner of supporting the cross girders was advantageous, as the height between the rail level and under side of the girder was very small. The observation of bad points, and the keeping in order of the bridge, was also rendered much less difficult by the avoidance of the numerous small hollow spaces of the box girders. This improvement suggested a still further advance; viz. the avoidance of all hollow spaces, the girder thus resulting being of the double T form. The top and bottom plates were united by means of a vertical rib and four angle irons riveted on. Such girders, when small, have since been generally made of one piece.

The top and bottom plates of the wrought-iron girder bridges hitherto noticed were connected by full panels, by means of which the strains of the bridge were conveyed to the flanges. From the experiments made on the Britannia bridge it was already evident that the bending of the vertical walls was the result of pressure, which worked downwards, but the experi-

menters do not appear to have been quite clear about the way in which these forces worked. Their nature had, however, been practically explained by means of the wooden suspension and lattice bridges of Long and Town. It was already known that these forces could be transmitted by means of single bars, and that therefore the vertical ribs could be replaced by the same. Rider constructed a bridge entirely of iron on this principle, which showed that he had already a considerable insight into the nature of those strains, as he constructed the struts and ties respectively of cast and wrought iron. An example of this construction is the bridge over the Royal Canal, for the railway between Dublin and Drogheda, made in the year 1845 (4). The webs of the girders of this bridge proved to be too weak, in consequence of the want of all stiffening ribs. This fault was corrected by James Barton, in 1855, in the construction of the lattice bridge over the Boyne at Drogheda. The railway was carried by means of four girders over the river, of which two were at one side of the bridge and two at the other. These girder pairs were connected by angle irons, riveted on diagonally, so that they were firmly braced together, and gave the necessary stiffness. Mr. Cubitt erected a similar bridge, with some improvements, over the Thames at Blackfriars, for the London, Chatham, and Dover Railway. The same engineer also constructed a bridge, on the already mentioned Warren system, over the Trent at Newark. This is superior to the lattice bridge. In consequence of greater simplicity, less iron is required.

To these girders with parallel booms succeeded those in which the upper member was in the arch form, viz. the bowstring girder bridges.

In the early part of the century, and up to 1850, Germany followed England, and simply copied its bridges as well as its railways. Among the early lattice bridges there constructed, which shows a considerable acquaintance with the real strains in those bridges, is that over the Saale at Grizena on the Magdeburg and Leipzig Railway. Numerous bridges of this sort were from this time constructed, most of them with very fine meshes, and, in consequence, an immense number of diagonals.

and with vertical stiffening ribs. A handsome one of this class occurs at Strasburg, over the Rhine. The diagonals of these bridges were all formed of flat iron. Owing to the small amount of rigidity possessed by them, Von Ruppert, when called upon to build a bridge over the Eipel, in north Hungary, left out the vertical stays, and made the diagonals in the half-cylinder form, and in order to spare material made the meshes at once much larger. The booms were cross formed, and the diagonals were riveted together at each crossing point with four rivets, in order to avoid displacement. This last arrangement was a fault in construction, as will be later explained. One span of this bridge is shown in Fig. 8.

This bridge builds the transition from the lattice bridges with small meshes and flat diagonals to those with wide meshes and round or cross-formed diagonals. The North German engineers soon followed up this improvement of the old fine-meshed lattice girders. The bending of the diagonals, together with the impossibility of following up theoretically the thus existing strains, induced them to have recourse to the lattice girder with wide meshes and diagonals of rigid section which were not fastened together at the crossing points. On this principle was the Flackensee bridge of the Niederschlesisch Märkische Railway (5) constructed. As will be seen in Fig. 9, the tie diagonals consist of flat iron, the struts of four angle irons, of which two and two are riveted together. At the crossing point the diagonals are only connected by one bolt. The bridge over the Mosel, at Coblentz, and that over the Nahe, at Bingen, are similar constructions.

Monié published, in the year 1858 (6), an "improved construction of lattice bridges," which he had patented. It consists in the avoidance of diagonals inclined in both directions, and in the employing inclined ties but vertical struts of a profile calculated to bear pressure. He, however, retained the system of riveting the diagonals to one another. This plan has been employed in the construction of a bridge at Allahabad, over the Jumna, on the Calcutta and Delhi Railway (7), and also for a bridge over the Old Rhine, at Griethausen. This riveting together of the diagonals was left out in the bridge over the

Rhine between Ludwigshafen and Mannheim, and it would appear that this principle is theoretically the clearest, and practically the most economical, of those employed in the construction of lattice or trellis girder bridges with parallel booms. Among the new bridges of this sort may be mentioned that over the Danube Canal at Vienna, between that town and Stadtlau. In this bridge all struts are of cylindrical form, each consisting of four pieces riveted together.

Von Pauli took out a patent in the year 1856 for a bowstring bridge, in which the girder flanges were so curved that the strain on them was the same in every part of the length when the bridge was loaded. The connecting of the two booms was effected by means of rigid vertical struts, and between these flat iron diagonals. The roadway is only connected with the vertical struts, not with the diagonals, and may be placed over, under, in the middle of the height of the girder. The one end of the girder is always carried on rollers, to allow for expansion and contraction. The first bridge thus constructed was that over the Isar, at Grossheselhöhe, on the Munich, Rosenheim, and Salzburg Railway (8). The top flange is in the box form, the bottom one consists of flat iron plates, the verticals are composed of four angle irons. The railway bridge at Mayence, over the Rhine (9), also executed on this principle, has thirty-two spans, the four middle ones being 342′ 6″. The height of the girder in the middle of these larger openings is 49′ 3″, in consequence of which the upper flanges of the girders on each side could be horizontally braced together. In this bridge the flange is of the double T-formed section, shown in Fig. 10, which also shows a view of half of one span.

Recent proposals have suggested constructing continuous girders on this principle, but as yet this has not been carried out. It appears that the advantages to be thereby obtained could not be very great, and the difficulties of construction must be considerably increased. For further information on the subject reference may be made to a lecture by Von Ruppert, at the fourteenth meeting of the German architects and engineers in Vienna, in the year 1864.

Having thus cursorily mentioned some of the principles on

which girder bridges have been constructed, together with some of the bridges where these principles have been employed, we arrive at that suggested by Schwedler. The comparison of the various systems will be postponed till the end of the paper.

Schwedler has endeavoured to unite the advantages of the girder with parallel flanges, and of that with the upper flange curved. The under boom is horizontal, the upper curved in such a manner that the diagonals can only be strained in tension in whatever way the bridge may be weighted, and also that the fewest possible number of diagonals are required, that is to say, cross diagonals are only necessary in some of the middle fields of the girder.

I will now proceed to explain the theoretical principles and formulæ on which the calculations for these bridges are based. My hearers will excuse me if I begin with somewhat simple formulæ with which they are all possibly acquainted; but as the following mode of calculation was first suggested by Schwedler, it almost belongs to the subject which is here treated, although it can be applied to any form of bridge. In substance these principles have been already published in English by Mr. William Humber, in his excellent treatise on 'Iron Bridge Construction.' If, however, I may be allowed to make a critical remark on so valuable a work, it appears to me that the following method is somewhat shorter, and shows also the connection between the vertical force, or, as he calls it, shearing force or strain, and the moment of strain in a somewhat clearer manner. In any case his formulæ appear in a different form, and it will be as well to explain from the beginning how the Schwedler formulæ are obtained. I have retained the German expression, vertical force, here in preference to shearing strain, inasmuch as the force referred to, although in girders with continuous webs it acts as a shearing force in the iron, in lattice girders only produces tension or pressure in the struts and ties of the bridge.

We will first consider the action of a single force acting on a beam. Let the force R act on the beam in Fig. 1, Plate 2. What is the action of this force at a section in the distance x from the point of application? Suppose in the plane of the section two equal and opposite forces R and $-$ R applied.

These forces, being equal and opposite, will nullify each other, and can therefore produce no increase of strain in the beam. The first-mentioned force R at A, taken with the $-$ R at the point of section, unite to produce a moment of strain tending to bend the beam equal to $R\,x$, and the force R at point of section remains as vertical force. The beam could therefore be actually cut in two at this point, and the remaining part of the beam would suffer the same strains as before, if, at the point of section, a moment of strain $R\,x$ and a vertical force R were there applied (see Fig. 2). A force $-$ P (Fig. 3), acting in the opposite direction at a distance a, gives similarly at the point of section a moment of strain $-P\,a$, and a vertical force $-$ P. When both these forces R and $-$ P act at the same time, the algebraical sum of the two moments, and the two vertical forces, will give us the united action at the point of section. The resulting moment of strain will therefore be $M = R\,x - P\,a$, and the vertical force $V = R - P$. Similarly are the resulting moment of strain and vertical forces for any number of forces, when those acting in one direction have the sign plus, those in the other minus,

$$M = \Sigma R x - \Sigma P a; \quad V = \Sigma R - \Sigma P.$$

We have therefore the rule:

The moment of strain at any section is equal to the algebraical sum of the moments of all forces to the left of the section in respect of the same, and the vertical force at the section is equal to the algebraical sum of these forces.

I have said to the left of the section. Whether we go to the left or right is immaterial, but as one must be chosen, I have chosen the left in the future calculations. It must also be evident that the forces acting on both sides of the section need not be considered. To make this clear, suppose a beam, as at Fig. 4, is weighted at various points. If the beam in this condition does not move, it is evident that at every point of it equilibrium must exist. If, therefore, at any point of section A I suppose the beam cut, and apply at this point forces exactly equal to the resultant of all former forces to the left of the section, it is evident that the equilibrium will not be disturbed,

and that the strain at the point of section and on the remaining part will be those which formerly existed. How to obtain such resultant has just been shown.

Let the moment of strain for Section I., Fig. 5, be $M = \Sigma R a - \Sigma P b$, and the vertical force $V = \Sigma R - \Sigma P$, then in respect of Section II. we shall have

$$M_1 = \Sigma R (a + x) - \Sigma P (b + x) + \Sigma S c - \Sigma Q d$$

for the action of any number of forces. S and Q are forces which act between the points of section, and c and d their distances from II. But

$$\Sigma R (a + x) = (R a + R x) + (R_1 a_1 + R_1 x) + (R_2 a_2 + R_2 x) + \ldots . \\ = \Sigma R a + x \Sigma R.$$

Similarly

$$\Sigma P (b + x) = \Sigma P b + x \Sigma P;$$

therefore

$$\Sigma R (a + x) - \Sigma P (b + x) = \Sigma R a - \Sigma P b + x (\Sigma R - \Sigma P) = M + V x;$$

therefore

$$M_1 = M + V x + \Sigma S c - \Sigma Q d; \quad V_1 = V + \Sigma S - \Sigma Q;$$

therefore, when single forces act on a beam (below the resistances of the supports, above the weights), and when we lay Sections I. and II. so that no force act between them, we shall have

$$M_1 = M + V x; \quad V_1 = V;$$

therefore between each two of the single forces the vertical force remains the same, and the moment changes in such a manner that, according to the laws of graphostatics, it may be represented by a straight line, which forms an angle with the axis of x equal to V. The formulæ show also that at the point of application of each single force V changes to the amount of the force, and according to our method of algebraical designation, becomes so much larger when the force is a reaction or resistance (from the supports), and so much smaller where the force is a weight. As V is the tangent of the angle of inclination of line which represents the moments, and as V suddenly changes its value at the point of application of each force, therefore the inclined line representing the moments

suddenly changes its direction at each such point. It descends less rapidly or mounts more quickly when the force is a reaction, the contrary when it is a weight. The line of moments rises, therefore, when the vertical force is positive (because the tangent of the angle is positive), and sinks when negative. The moment of strain is, therefore, a maximum or minimum in those places where the vertical force goes through zero, or from plus to minus. The Fig. 6, with the diagram of vertical forces and diagram of moments, will render this clearer.

If the beam carries an equally distributed load p for each unity of length, the load between the two sections will be $p\,x$ (see Fig. 7), therefore in analogy with the foregoing $V_1 = V - p\,x$. The load $p\,x$ has its centre of gravity in the middle, therefore we have also $M_1 = M + V\,x - p\,x\frac{x}{2}$ or $M_1 = M + V\,x - \frac{p}{2}x^2$. This is the equation of a parabola. Therefore, for a beam with equally distributed load, the vertical forces may be represented by an inclined line, and the moments of strain by a parabola with vertical axis and vertex at the top (see Fig. 8). I will give one example of the application of these formulæ. Let the beam in Fig. 8 have an equally distributed load p per running foot. Let the length be l feet. Each support must carry half the weight, therefore $\frac{p\,l}{2}$. We have therefore, at the left-hand point of support, $V = \frac{p\,l}{2}$, M is here, of course, zero, as we have no lever arm. If we proceed to a point at a distance x from the support, we have, according to the above formulæ, $V_1 = \frac{p\,l}{2} - p\,x$, and $M_1 = \frac{p\,l}{2}x - \frac{p}{2}x^2$. This equation will have its greatest value when $x = \frac{l}{2}$, therefore in the middle we have $M = \frac{p\,l^2}{8}$, which we measure off as height in the middle to any convenient scale. We can now construct our parabola, having this middle point as vertex, and knowing that it goes through two points of support. For $x = \frac{l}{2}$, $V = 0$,

therefore the line representing the vertical forces crosses the axis of x here, and at $x = l$, the ordinate will be $V = -\frac{pl}{2}$.

Having these principles clearly before us, we now proceed to consider the action of movable loads on a bridge. In practice, the girders of a bridge are not weighted with an equally distributed load, but at the points where the cross girders are attached. Let Fig. 9 represent such a girder. The various divisions of the girder between each cross girder may be called bays, and the bay before the x'th cross girder shall be called the x'th bay. Let G and G_1 be the resulting forces of all loads on each side of the section in the x'th bay, g and g_1 respectively their distances from the supports. R is the reaction on the left side of the bridge. As the bridge is in equilibrium, the moment of all forces, with regard to any given point, must be equal to 0, therefore, regarding the right support as axis, we have

$$R\,l - G\,(l - g) - G_1\,g_1 = 0, \quad \text{or } R = \frac{G\,(l - g) + G_1\,g_1}{l}.$$

Therefore we have, according to the above formulæ, if V_x indicates the vertical force in the x'th field,

$$V_x = R - G = \frac{G\,(l - g) + G_1\,g_1 - G\,l}{l} = \frac{G_1\,g_1 - G\,g}{l}. \qquad [1]$$

Further, if M_x is the moment at the x'th point, we have $M_x = Rx - G\,(x - g)$, therefore putting in the above-found worth for R,

$$M_x = \frac{G\,(l - g)\,x + G_1\,g_1\,x - G\,l\,(x - g)}{l} = \frac{G\,g\,(l - x) + G_1\,g_1\,x}{l}. \qquad [2]$$

These two equations show, 1st, that the vertical force is a maximum when the bridge to the right of the section is loaded in maximo, left of the section in minimo. When the contrary is the case, viz. the maximum load on the left the minimum on the right, we have a minimum vertical force or a negative maximum vertical force.

2nd, that the moment of strain is the greatest with a maximum load on the whole bridge.

Those parts of the construction therefore which depend upon

the vertical force will be strained in maximo by a partial loading of the bridge in the manner described, those which depend on the moment by a maximum loading of the bridge.

The point, in which the vertical force goes through zero or changes from + to − or *vice versâ*, may be called the apex (because the moment of strain is here a maximum, and this is therefore the apex of the curve representing the latter), and as this point varies according to the amount of load on the bridge, it is therefore a *moving apex*. We shall have, as before mentioned, a maximum vertical force, at any section, when the bridge is loaded on the right of the same in maximo, on the left in minimo. If now we move our section (always so loaded that the vertical force is a maximum) till we arrive at a point where the maximum vertical force is = 0, we shall arrive at one extreme of the path of the moving apex; by loading our bridge in the opposite manner, and moving our section the opposite way, we shall arrive at the other end of the path of the moving apex. The distance between these two points may be called the traject of the moving apex. At the left hand of the bridge will remain a certain distance, from the supports to the end of the traject, in which the vertical force is always positive, in whatever way the bridge may be loaded; in the same way at the right end a similar distance where it is always negative. To explain this out of the formulæ we have V maximum = 0 as the condition for finding the point referred to. Put this in the equation and we have $G_1 g_1 - G g = 0$. Let us suppose p to be the weight of the bridge pro unity of length, as in Fig. 10, q the running load on the same, also pro unity of length. Let x be the distance of the end of the traject from the centre of the bridge. We have then

$$V_x = \frac{G_1 g_1 - G g}{l} = \frac{(p+q)\left(\frac{l}{2}+x\right)^2 - p\left(\frac{l}{2}-x\right)^2}{l} = 0.$$

This gives

$$x = \frac{l}{2}\left(1 + 2\frac{p}{q} - 2\sqrt{\frac{p}{q}+\frac{p^2}{q^2}}\right).$$

x can therefore only become $=\frac{l}{2}$, that is to say the vertical force can only become zero at the ends of the bridge when p or its own weight = 0.

Having obtained the formulæ [1] and [2], which supply the means of calculating the maximal moments and vertical forces, the formulæ employed for determining the strains on the different members of the girder may now be developed, the moment and vertical force being given. Let the lengths and angles of the various members of one bay of the girder be as shown in Fig. 11; let the strains in these members be those shown in Fig. 12, and denoted by the capital letters A, D, T, P. The necessary equations are now easy to find, being calculated on the principle of moments, a suitable point being in each case chosen as axis. If we suppose that all the forces in the parts cut by any section are forces of tension, then in the results plus will signify tension, minus pressure. Let the section be supposed immediately on the right-hand side of the point x and vertical, and let the point B (Fig. 11) be the axis, then is the necessary condition of stability for the bridge

$$0 = M_x - A_{x+1}\, y_x ,$$

or

$$A_{x+1} = \frac{M_x}{y_x} \text{ (see Fig. 13).} \qquad [1]$$

Section now on the left side of point x, and vertical, axis point x, we have

$$0 = M_x + T_x \cos. \alpha_x\, y_x, \quad \text{or} \quad T_x = -\frac{M_x}{y_x} \cdot \frac{1}{\cos. \alpha_x}; \quad \text{but} \quad \frac{1}{\cos. \alpha_x} = \frac{t_x}{\Delta x},$$

therefore

$$T_x = -\frac{M_x}{y_x} \cdot \frac{t_x}{\Delta x}. \qquad [2]$$

We obtain also, on supposing the section in the middle of the x'th bay and vertical, and setting the sum of the vertical forces $= 0$,

$$0 = V_x - D_x \sin. \beta_x + T_x \sin. \alpha_x ,$$

which gives (out of the figure and equation [2])

$$D_x = \frac{d_x}{y_{x-1}} \left(V_x - \frac{M_x}{y_x} \cdot \frac{\Delta\, y_x}{\Delta\, x} \right). \qquad [3]$$

On supposing the section parallel the diagonals as in Fig. 14 we have

$$V_x + P_{x-1} + T_{x-1} \sin. \alpha_{x-1} = 0,$$

therefore, putting in the above-found value for T,

$$P_{x-1} = -V_x + \frac{M_x - V_x\, \Delta\, x}{y_{x-1}} \cdot \frac{\Delta\, y_{x-1}}{\Delta\,(x-1)}. \qquad [4]$$

If the diagonals are in the other direction the formulæ may be developed in a similar way. They are as follows:

$$A_x = \frac{M_x}{y_x}; \tag{5}$$

$$T_{x+1} = -\frac{M_x}{y_x} \cdot \frac{t_{x+1}}{\Delta (x+1)}; \tag{6}$$

$$D_x = -\frac{d_x}{y_{x-1}} \left[V_x - \frac{M_x}{y_x} \cdot \frac{\Delta y_x}{\Delta x} \right]; \tag{7}$$

$$P_x = V_x - \frac{M_x}{y_x} \quad \frac{\Delta y_{x+1}}{\Delta (x+1)}. \tag{8}$$

Equations [1], [2], [5], and [6] show that the upper flange is always subject to compression, the under one to tension, and that they suffer a maximum strain by a maximum load on the bridge.

Equations [3] and [7] are equal in their absolute value, but we have plus in the first case, minus in the second. If, therefore, a diagonal in the first direction is compressed, and were replaced by one in the second, this latter would be strained in tension, and *vice versâ.* The proportion in which the strains stand to one another will be directly as their lengths. It may also be deduced from these equations that every weight to the right of a diagonal in the direction of that in Fig. 11 produces tension in the same. Therefore such diagonals suffer the greatest amount of tension with a maximum load on the bridge to the right of them, a minimum to the left, that is to say when the vertical force is a maximum. The contrary is the case for the diagonals in the contrary direction. If, therefore, we wish to employ diagonals only as ties, those bays in which the vertical force can only be positive will have ties in the direction of those in Fig. 11, those bays in which the vertical force can only be negative will have ties in the contrary direction; between these points both diagonals will be required.

Out of [4] and [8] may be proved that the vertical struts will sustain a maximum pressure when the tension in the diagonals is a maximum. Having thus given the principles and obtained the formulæ by which the strains on any part of a girder bridge may be obtained, we now proceed to the "Schwedler" bridge.

Out of the formulæ [3] it is evident that when Δy_x does not exist, that is to say when the two flanges are parallel, we shall

have a maximum tension in the diagonals, other things being equal; and as long as Δy_x is positive, that is to say when the boom rises the strain will be lessened; and if Δy_x becomes large enough the strain may become negative, even with a positive vertical force, that is to say the diagonal would be compressed. If, therefore, we wish to employ only such diagonals as sustain tension, we shall have to put into each bay, where the diagonal may be compressed with a minimal V_x, a diagonal in the opposite direction also. In the bowstring girder, in which the upper boom is parabolic, diagonals are required in every field in both directions. Schwedler's object has been to avoid this necessity and therefore at the ends of the bridge, where the vertical force is only positive or only negative—in other words, that part of the bridge outside the traject of the moving apex—he has regulated the Δy_x in such a way that the diagonals can never be compressed, so, therefore, that only one diagonal is required in these bays. To effect this object it will be necessary to make the strain on these diagonals, when suffering a minimum of tension, or, as before explained, when the vertical force is a minimum, equal to 0, so that they can under no circumstances be compressed. We have therefore, as condition for those bays of the bridge up to the ends of the traject of the moving apex, measuring from each end, the equation

$$D_x = 0 = \frac{d_x}{y_{x-1}}\left(V_x - \frac{M_x}{y_x} \cdot \frac{\Delta y_x}{\Delta x}\right)$$

where V_x is V minimum for this point and M_x is the corresponding moment. It follows, therefore, that

$$\frac{\Delta y_x}{y_x} = \frac{V_x \Delta x}{M_x}.$$

The curve given by this equation would rise as long as V_x is positive, and fall with a negative V_x. It would therefore rise as far as the end of the traject of the moving apex, and then fall, as shown in curve 1, Fig. 15. The same may be said for the diagonals in the opposite direction when we have a maximum V_x. (See curve 2, Fig. 15.) In the middle part, between the summits of these curves, where the vertical force may be

either positive or negative, according to the mode of loading the bridge, the upper flange is put in horizontal and the diagonals are required in both directions. It must not be forgotten that the two diagonals are only required when we wish merely to have diagonals which are subject to tension. When therefore in the middle bays, where we have diagonals both ways, and those in one direction would suffer pressure by any particular loading of the bridge, they merely give way a little, being constructed of flat iron which easily bends, and allow the diagonals in the opposite direction to receive the strain as if they were altogether absent.

The height of the middle of the girder must be left to the engineer's judgment, and is generally from $\frac{1}{8}$th to $\frac{1}{10}$th of the span of the bridge. As before mentioned, this height is retained till the end of the traject of the moving apex. We have therefore y_x given us for the bay where the curve begins. There remains in the equation of the Schwedler girder only Δy_x unknown. The solution of the equation will give this, and we have then $y_{x-1} = y_x - \Delta y_x$. By this means we have y_{x-1}, which we put into the above equation. We solve this new equation, and thus have y_{x-2}, and so further. Having thus calculated the ordinates or verticals of our bridge, the lengths t_x and d_x (Fig. 11) can be reckoned, or measured from the drawing. We now calculate the maximum strains in each part, according to the formulæ [1] to [9].

The curve in which the upper parts of the girder lie may be found on setting $\Delta x = d x$; $\Delta y_x = d y$. The equation becomes $\frac{d y}{y} = \frac{V d x}{M}$. Upon integration this becomes $y = \frac{C x (l - x)}{p l + q x}$, where C is a constant, q the running load, and p the weight of the bridge pro unity of length.

Having thus explained the mode in which the calculations are effected, this will be a suitable place for comparing this system with those previously existing. In this comparison I shall avoid mention of bridges of wrought and cast iron together, having at the commencement of the paper explained the reason of the abandonment of the same. It would appear, as before hinted, that the first bridges entirely of wrought iron were con-

structed, in England, between 1846–1848, and that these were immediately followed in North America by bridges on Town's system. The box girders, as at first made in England, were soon given up for trellis and lattice girders, for reasons already given, except for bridges of very small spans. The lattice girder bridges were first made with webs, consisting of flat bars at equal and small distances apart, and of equal strength throughout. This, in consequence of a better understanding of the theory of strains, especially with regard to the vertical force, was followed, more particularly in Germany, by lattice girders with large meshes composed of bars of more rigid profile, which, however, were bolted together, thereby preventing any precise calculation of the strains on each of them, as by the smallest change in the form of the bridge, such as that occasioned by partial loading of the same, it is impossible to know in what way these diagonals may be affected. The next step was the making of the diagonals subject to tension out of flat iron, as at first, and retaining the rigid form of section merely for the struts. The next advance was that which brought bridges to their present state of development, in which the diagonals were not bolted to each other (or only by one bolt, to prevent their jarring on the passing of a train), by which means all bending strains were avoided. Among the systems of this sort, those in which the struts are vertical (and in consequence the ties sloping) are to be preferred, inasmuch as vertical struts are the shortest and least liable to flexure. In this form is the girder with top and bottom flanges parallel, a perfectly correct and theoretically complete construction. It has, however, the disadvantage of requiring much more iron at the ends than those in which the upper flange is curved. The Warren girder has perhaps an advantage in appearance over those with vertical struts. The bowstring girders on the Pauli system show also a perfect understanding of the theory of bridge construction. Their only advantage would appear to be that the section of the flanges is the same throughout. As, however, the angle irons, &c., of which the flange is composed must be jointed frequently, whether we retain those of the same size throughout or vary their size, as we must do by any other form of girder,

would not appear to be very important. The American engineer Roebling has carried out the system of girders with both flanges curved in the parabolic form in various bridges in America. The girders, however, with the under boom straight have the advantage over these and the Pauli bridge of admitting an easy connection of the roadway with the under boom. Among the girders of this class the "Schwedler" bridge would appear to have advantages over the other systems. As before mentioned, the girders with parallel flanges require more iron than those with the upper flange arched. When the latter is of the form of a parabola every bay requires a double diagonal, many of which are saved in the Schwedler girder. The very acute angle which the parabolic line forms with the under horizontal boom renders the connection of the two very difficult. This difficulty is lessened considerably in the "Schwedler" bridge, as the angle is always much more obtuse. Figs. 11 and 12, Plate 1, show the difference between the parabolic and "Schwedler" bridge. Fig. 12 represents a railway bridge designed by the author for a span of 150 feet.

Having explained what would appear to be the advantages of the "Schwedler" bridge over other iron girder bridges, I will now give an example of one of these. A simple one has been chosen, in order to render the principle more evident. The bridge elected for this purpose is that over the Oder, at Breslau (10).

This bridge has five spans of 76 (Prussian) feet each over one arm of the river, two spans of the same length over the other. The piers are 8 feet thick. Their direction was chosen to suit that of the river, those in the one bridge thus forming an inclination with the bridge axis of 25 : 8, that in the other 25 : 4. The piers are founded on béton and built of stone. A general view of the bridge is shown on Plate 3. All the girders are similar to the one shown on this Plate, only requiring double diagonals in one bay. This is owing to the heavy weight of the roadway in comparison with the running load. In railway bridges, where the roadway is much lighter and the running load heavier, more are generally required. (See Fig. 12, Plate 1.) Particular attention may be drawn to the form of section of the upper and under booms. It admits a thorough

examination whenever required. Any defects can therefore be immediately discovered, and the thorough painting of the bridge is rendered easy. One end is placed on rollers to allow for expansion and contraction, the construction being similar to that of the Jumna bridge, illustrated in Humber's 'Bridge Construction.' In the calculation for the under boom the rivets have been allowed for, viz. one in each flange of every angle iron. In order to render the girder rigid the two vertical plates of the upper boom are placed at distance of 14 inches apart and united by lattice work of flat iron. The jointing of the vertical plates has been made, as shown on Plate 3, by means of half-inch plates, placed in the space between the vertical plate and the inside angle irons. The jointing of the inside angle irons is effected in the middle of the bays, the vertical flange being jointed by means of a plate in the space just mentioned, the horizontal flange in the half-inch space between the angle irons. The plates for the jointing of the vertical flange of the exterior angle irons are placed in the plane of the vertical plate, where it is broken in order to be jointed itself. (See detail of joint at point C, Plate 3.) The diagonals consist of two bars, half an inch thick, and of the necessary breadth. The verticals suffer only tension and consist of four angle irons braced, as shown on the section, Plate 3. The cross girders are of ordinary form and full web, three-eighths of an inch thick. The footpaths are placed outside the main girders. The horizontal bracing is of usual construction. According to the calculations the iron is strained in maximo with 5 tons to the square inch. The roadway is paved, and supported on cast-iron plates. The weight of the bridge is, for each foot of length:

For the main girders	8 cwt. wrought iron.
„ roadway	8 „ „
„ end supports and plates for the roadway..	8½ cwt. cast iron.

Among other bridges on this system may be mentioned the railway bridge also at Breslau, and the bridge over the Elbe, on the Berlin-Lehrter Railway line.

REFERENCES.

For further particulars of the bridges to which the reference numbers are attached, the following works may be referred to:

1. 'Allgemeine Bauzeitung,' Vienna, 1846, p. 275.
2. " " " 1852, p. 166.
3. Molinos et Pronnier, 'Traité de la Construction des Ponts Métalliques,' p. 332.
4. 'Zeitschrift für Bauwesen,' Berlin, 1857, p. 215.
5. " " " 1859, p. 37.
6. " " " 1858, p. 277.
7. 'A Complete Treatise on Cast and Wrought Iron Bridge Construction,' Humber.
8. 'Sammlung eiserner Brücken Constructionen,' Klein, Stuttgart, 1864.
9. 'Eisenbahnbrücke über den Rhein bei Mainz,' Mainz, 1863.
10. 'Zeitschrift für Bauwesen,' Berlin, 1868, p. 158. The calculations are here given in detail. Most of the other bridges built on the "Schwedler" system have also appeared in this journal from time to time.

Plate 1.

. 8.

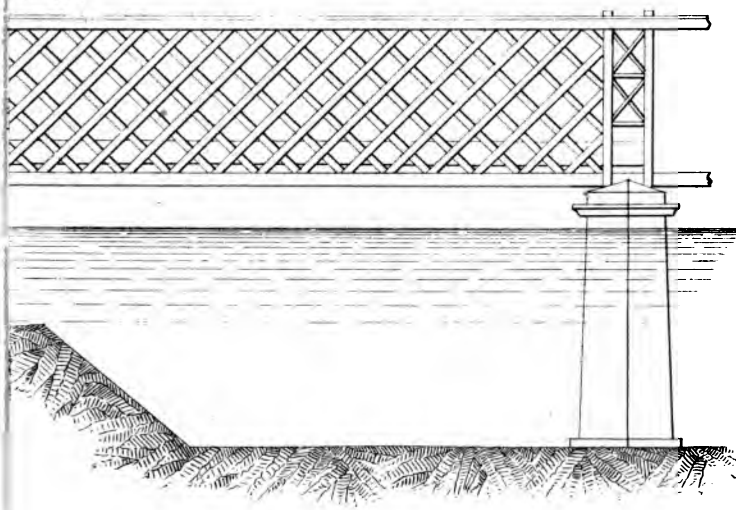

Fig. 10.

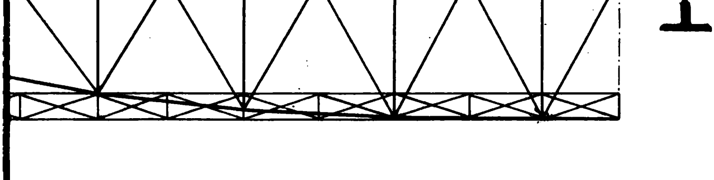

Fig. 12.

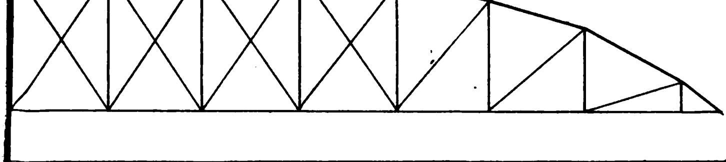

Plate 2.

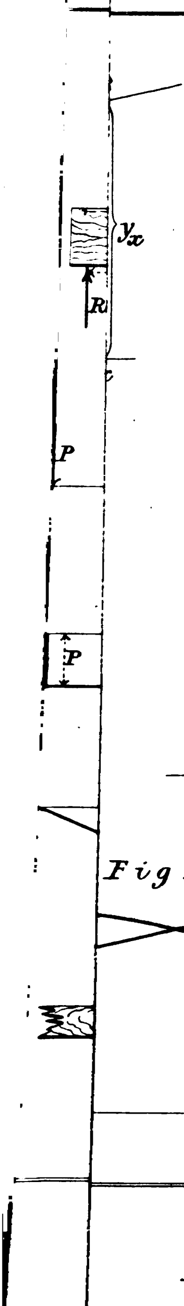

Fig. 12.

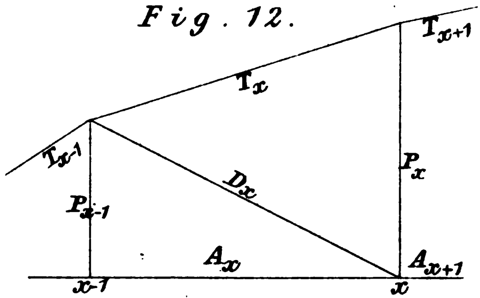

Fig. 14.

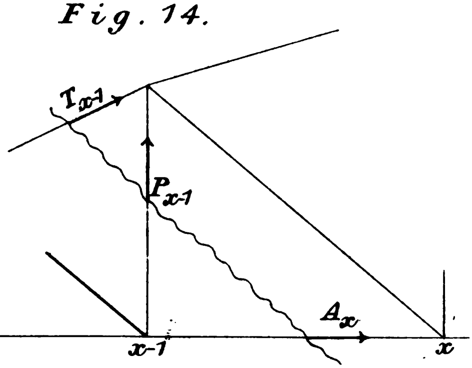

Fig. 15.

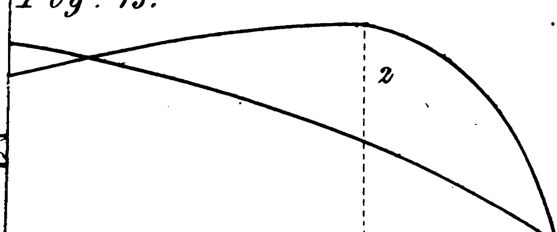

Plate 3.